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Climate Change Risk in Finance: a credit risk approach
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Climate Change Risk in Finance

A Credit Risk Approach

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Lecture's focus

The goal

The aim of this lecture is to focus on:

- the modelling of climate change risk (transition risk component)
- the modelling the impact of climate risk into the credit risk of a corporation

To do so we consider a multidimensional framework, where:

- hazard rate is stochastic and multidimensional in the state variables;
- the climate change risk is a component of the hazard rate, which states for the indicator of the transition;
- the info about the transition risk are captured by the spot prices of the emission allowances in the EU-ETS market.

Reasons and Structure

The research question is justified by:

- Climate change is the great challenge of our time and a deep discussion about the effect of climate change on financial market allow us to focus on the stability of our economic system;
- Few technical contribution in literature;
- Climate change directly affects the firm's ability in making profits and then in repaying their liabilities; therefore we dedicate our attention to the impact on credit risk;

The structure of the lecture will be:

1. Review of the Eu emission trading scheme and the related empirical evidences;
2. Credit Risk modelling with stochastic hazard rate;
3. Credit Risk modelling with climate change impact in view;
4. Empirical experiments and simulation analysis.

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The Green Swan

The green swan is a climate change-related event that will happen in the future and may cause instability and economic crisis, but whose timing is uncertain and far in the future.

Climate Risk can be decomposed into:

- Physical Risk: it is associated to the happening of extreme weather events (long periods of drought, high frequency of hurricanes, high temperatures...)
- Transition Risk: it is associated to the switch from a carbon intensive production to a low carbon one

The main issue in the future for financial actors will be how to identify and measure the climate change risk and the strategy to implement in order to minimize the cost of the transition to a low carbon economy.

Credit risk and Carbon risk

Empirical evidence in support of their relationship:

- Jung, Herbohn and Clarkson (2016): they perform a regression between the cost of debt and the carbon intensity plus other relevant variables finding that firms with poor environmental performance (high carbon intensity) pay more in term of interest rates than firms with lower carbon intensity.
- Bauer and Hann (2010): they show that firms with environmental concerns have less reputational, legal and regulatory costs, and then also lower financing costs and higher ratings.
- Capasso, Gianfrate and Spinelli (2010): they perform a regression using as dependent variable the distance to default and as independent variables the carbon intensity (plus other relevant variables) and find that the distance to default increases as the carbon intensity increases and decreases as the carbon intensity decreases.

EU-Emission Trading System

In order to facilitate the transition from a carbon intensity economy to a low carbon one (assuring for low cost and standard procedure):

- 2005: according to Kyoto Protocol, EU created the EU-ETS, a financial market where emission allowances were exchanged between firms.
- The emission allowances allow the owner to emit 1 tonne of CO2.
- The market of the emission allowances is a cap and trade system: each year the authority auctions a predetermined number of emission allowances then during the year the certificates are exchanged in the market. In the next year the authority check if the total number certificates owned by every operator covers the total emissions of the firm during the year, and the certificates used for covering the emission are cancelled. If the firm cannot cover its total emissions with certificates it must pay a penalty which is 100 euros per tonne.

EU-Emission Trading System

EU-ETS transforms the greenhouse gases into a commodity whose market price is equal to the marginal abatement cost of the greenhouse gases.
We identify 4 phases during which the regulators has changed the regulations of the markets and increased the number of sectors that must operates in them:

- phase 1: 2005- 2007,
- phase 2: 2008-2012,
- phase 3: 2013-2020,
- phase 4: 2021-2030.

An EFFICIENT cap and trade system assures for working properly (GHGs are reduced year by year). It happens if the price of the certificates is equal (or very close) to the marginal abatement cost (in equilibrium) of GHGs.

EU-Emission Trading System

In an efficient ETS, to fulfill any need of production a company can choose to invest into a technology that abates emissions for producing goods or to buy emission allowances to cover the production using the old carbon intensive technology.:

- if abatement costs are lower than the price of the emission allowances: managers will take the investment opportunity;
- if abatement costs are greater than the price of the emission allowances: managers will buy the certificates, so pushing up the price of the allowances and making the low carbon technology more appealing again.

An Efficient ETS transfers in prices the information about the transition risk (high risk if the abatement costs are high, low risk if they are low).

EU-ETS and stock markets

Are stocks prices of the companies belonging to sectors covered by the EU-ETS influenced by the prices of the emission allowances?

- Bassen and Rothe (2009): they found out for European utilities that for both, carbon intensive and low carbon companies, the cost of capital can be explained (with a Fama-French approach) also by a carbon risk factor expressed as the excess returns of carbon futures (futures on emission allowances certificates) against the risk-free rate. The carbon factor results to be high for carbon intensive companies and low for low carbon ones.

EU-ETS and stock markets

- Tian et al. (2016): they conducted an analysis on companies operating in the electricity production distinguishing between clean energy producers and carbon intensive energy producers finding that clean energy stock prices are positively related to the spot price of the emission allowances while stock prices of carbon intensive producers are negatively related to the spot price of the emission allowances.
- Oberndorfer (2009): conducted an analysis on the relationship between stock prices of electricity firms and emission allowances finding that the effects on stock prices differs between corporations belonging to different geographic areas.

Efficiency of EU-ETS

Is EU-ETS efficient?

- Phase 1: It has been proved that it wasn efficient (see Hintermann, 2009 and Hintermann, 2012)
- Phase 2: results are not so clear, because there are papers supporting the efficiency while others disagree on this point finding that the main driver are the economic activity, regulatory acts and other unobservable variables (see Andj et al. 2020).
- Phase 3: a frequency analysis is performed using a VAR time series approach and it is showed that the efficiency of the market has increased and that the spot prices of the emission allowances are starting to incorporate the information about the marginal abatement costs (see Lovcha, Perez-Laborda, and Sikora, 2019).

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Affine Models

Let r_t be the short rate, then its dynamic is the following:

$$dr_t = \mu(t, r_t)dt + \sigma(t, r_t)dW_t$$

where W_t is a Brownian motion.

Affine term structure

The term structure $\{p(t, T); 0 \leq t \leq T\}$, where $p(t, T) = F(t, r_t, T)$, is said to be affine iff $F(t, r_t, T) = \exp\{A(t, T) - B(t, T)r_t\}$, where $A(t, T)$ and $B(t, T)$ are deterministic functions.

Affine Models

Let $T > 0$, hence $F(t, r_t) \in \mathbb{C}^{1,2}([0, T] \times Z)$ is solution of the boundary problem on $[0, T] \times Z$

$$\begin{cases} \partial_t F(t, r_t) - r_t F(t, r_t) + \mu(t, r_t)\partial_r F(t, r_t) + \frac{1}{2}\sigma^2(t, r_t)\partial_r^2 F(t, r_t) = 0 \\ F(T, r) = \phi(r) \end{cases}$$

This implies that $M(t) = F(t, r_t) \exp\{-\int_0^t r_u du\}$ is a local martingale which becomes a true martingale if one of the following holds:

- $\mathbb{E}^{\mathbb{Q}}[|\partial_r F(t, r_t) \exp\{-\int_0^t r_u du\}\sigma^2(t, r_t)|] < +\infty$
- M is uniformly bounded.

In this case, we have:

$$F(t, r_t) = \mathbb{E}^{\mathbb{Q}}[\exp\{-\int_t^T r_u du\}\phi(r_T)|\mathcal{F}_t],$$

where $\mathbb{Q}$ stands for the risk neutral measure.

Affine Models

How must we define $\mu(.,.)$ and $\sigma(.,.)$ in order to get an affine term structure?

The conditions are the following (see Bjork, 2009):

$$\begin{cases} \mu(t, r_t) = \beta(t) + \alpha(t) r_t \\ \sigma(t, r_t) = \sqrt{\xi(t) r_t + \psi(t)} \end{cases}$$

Therefore if $A(t, T) = A(T - t), B(t, T) = B(T - t)$ and $F(T, r_T) = 1$, then:

$$\begin{cases} \partial_t F = F(\partial_t A - r_t \partial_t B) \\ \partial_r F = -FB \\ \partial_r^2 = B^2 F \end{cases}$$

Affine Models

Consequently the PDE can be decomposed into two backward SDEs, i.e.

$$\begin{cases} \partial_t A = \beta(t)B - \frac{\psi(t)}{2}B^2 \\ A(0) = 0 \end{cases}$$

$$\begin{cases} \partial_t B = -1 - \alpha(t)B + \frac{\xi(t)}{2}B^2 \\ B(0) = 0 \end{cases}$$

whose solutions allow to recover the zcb prices.

Affine Models: Vasicek model

The $\mathbb{Q}$ dynamics of r_t is a Ornstein-Uhlenbeck process:

$$dr_t = k(\theta - r_t)dt + \sigma dW_t$$

Hence for $\alpha(t) = -k, \beta(t) = k\theta, \psi(t) = \sigma^2, \xi(t) = 0$, the previous SDEs become linear whose solutions are:

$$\begin{aligned} A(t,T) &= \left(\theta - \frac{\sigma^2}{2k^2}\right)(B(t,T) - (T-t)) - \frac{\sigma^2}{4k^2}B(t,T)^2 \\ B(t,T) &= \frac{1}{k}\left(1 - \exp\{-k(T-t)\}\right) \end{aligned}$$

Affine Models: CIR model

The $\mathbb{Q}$ dynamics of r_t is a square root process:

$$dr_t = k(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t$$

Hence for $\alpha(t) = -k, \beta(t) = k\theta, \psi(t) = 0, \xi(t) = \sigma^2$, the previous SDEs are not both linear since the second one is a Riccati one. The solutions are:

$$\begin{aligned}
A(t,T) &= \frac{2k\theta}{\sigma^2}\log\left\{\frac{2h\exp\{(k+h)(T-t)/2\}}{2h+(k+h)(\exp\{(T-t)h\}-1)}\right\} \\
B(t,T) &= \frac{2(\exp\{(T-t)h\}-1)}{2h+(k+h)(\exp\{(T-t)h\}-1)} \\
h &= \sqrt{k^2+2\sigma^2}
\end{aligned}$$

Credit Risk: Intensity-based models

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let $\mathcal{F}_t = \sigma(\{\psi_t : s < t\})$ be the filtration generated by some observed background process. Let the random time $\tau > 0$ a.s. on $\mathcal{F}$ and be $Y_t = \mathbf{1}_{\tau \leq t}$ the associated jump indicator and $\mathcal{H}_t = \sigma\{\mathbf{1}_{\tau \leq s} : s \leq t\}$ the filtration generated by Y_t, then the general filtration for our purpose is defined as

$$\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t$$

and τ resulted to be a stopping time with respect to $\mathcal{G}_t$ and $\mathcal{H}_t$ but not necessarily to $\mathcal{F}_t$.

Credit Risk: Intensity-based models

$\mathcal{F}_t$-conditional hazard process of τ

A random time τ is said to be doubly stochastic if there exist an $\mathcal{F}_t$ adapted and positive process γ_t, such that $\Gamma_t = \int_0^t \gamma_s ds$ is strictly increasing and finite for every $t > 0$ and such that, for all $t \geq 0$,

$$\mathbb{P}(\tau > t | \mathcal{F}_\infty) = e^{-\int_0^t \gamma_s ds}$$

Explicit construction of a doubly stochastic hazard process:

- X is an exponential rv on $(\Omega, \mathcal{F}, \mathbb{P})$, independent of $\mathcal{F}_\infty$, i.e. $\mathbb{P}(X \leq t | \mathcal{F}_\infty) = 1 - e^{-t}, t \geq 0$
- $\gamma_t > 0$, $\mathcal{F}_t$ adapted, such that $\Gamma_t = \int_0^t \gamma_s ds$ is strictly increasing and finite for every $t > 0$
- define $\tau = \inf\{t \geq 0; \Gamma_t \geq X\}$, where Γ_t is the cumulative hazard process of τ.

Credit Risk: Intensity-based models

If τ is a random time such that $\mathbb{P}(\tau > t|\mathcal{F}_t) > 0, \forall t$ it holds true that for every $\mathcal{G}_t$-measurable X, $\exists \tilde{X}, \mathcal{F}_t$-measurable such that $X\mathbf{1}_{\tau>t} = \tilde{X}\mathbf{1}_{\tau>t}$. Therefore:

$$\mathbf{1}_{\tau>t}\mathbb{E}[X|\mathcal{G}_t] = \mathbb{E}[\mathbf{1}_{\tau>t}X|\mathcal{G}_t] = \tilde{X}\mathbf{1}_{\tau>t}$$

$$\mathbb{E}\{\mathbb{E}[\mathbf{1}_{\tau>t}X|\mathcal{G}_t]|\mathcal{F}_t\} = \mathbb{E}\{\tilde{X}\mathbf{1}_{\tau>t}|\mathcal{F}_t\}$$

$$\mathbb{E}[\mathbf{1}_{\tau>t}X|\mathcal{F}_t] = \tilde{X}\mathbb{E}\{\mathbf{1}_{\tau>t}|\mathcal{F}_t\}$$

$$\rightarrow \tilde{X} = \frac{\mathbb{E}[\mathbf{1}_{\tau>t}X|\mathcal{F}_t]}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

Credit Risk: Intensity-based models

Moreover if τ is doubly stochastic with hazard process γ_t, $\mathbb{P}(\tau > t|\mathcal{F}_t) = e^{-\int_0^t \gamma_s ds}$, hence if X is $\mathcal{F}_T$-measurable:

$$\begin{aligned}
\mathbb{E}[\mathbf{1}_{\tau>t}X|\mathcal{G}_t] &= \mathbf{1}_{\tau>t}e^{\int_0^t \gamma_s ds}\mathbb{E}\{\mathbf{1}_{\tau>t}|\mathcal{F}_t\} \\
&= \mathbf{1}_{\tau>t}e^{\int_0^t \gamma_s ds}\mathbb{E}\{\mathbb{E}[\mathbf{1}_{\tau>t}|\mathcal{F}_T]|\mathcal{F}_t\} \\
&= \mathbf{1}_{\tau>t}e^{\int_0^t \gamma_s ds}\mathbb{E}[Xe^{-\int_0^T \gamma_s ds}|\mathcal{F}_t] \\
&= \mathbf{1}_{\tau>t}\mathbb{E}[Xe^{-\int_t^T \gamma_s ds}|\mathcal{F}_t]
\end{aligned}$$

Under the assumption that τ is doubly stochastic under $\mathbb{Q}$ with background filtration $\mathcal{F}_t$ and hazard process γ_t and stating $R_t = r_t + \gamma_t$, we have:

$$\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\tau>T}e^{-\int_t^T r_s ds}X|\mathcal{G}_t] = \mathbf{1}_{\tau>t}\mathbb{E}^{\mathbb{Q}}[Xe^{-\int_t^T R_s ds}|\mathcal{F}_t]$$

where we apply the previous equality to $e^{-\int_t^T r_s ds}X$ instead of X.

Credit Risk: Intensity-based models

Given the value of a claim at the default time τ, Z_τ, we have:

$$\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{t<\tau\leq T} e^{-\int_t^\tau r_s ds} Z_\tau | \mathcal{G}_t] = \mathbf{1}_{\tau>t} \frac{\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\tau>t} Z_\tau e^{-\int_t^\tau r_s ds} \mathbf{1}_{\tau\leq T} | \mathcal{F}_t]}{\mathbb{Q}(\tau > t|\mathcal{F}_t)}$$

where $\mathbb{Q}(\tau \leq t|\mathcal{F}_T) = 1 - e^{-\int_0^t \gamma_s ds}$ and the conditional density is $f_{\tau|\mathcal{F}_T}(t) = \gamma_t e^{-\int_0^t \gamma_s ds}$. Therefore we have:

$$\begin{aligned}
& \mathbb{E}^{\mathbb{Q}}\{\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\tau>t} e^{-\int_t^\tau r_s ds} Z_\tau \mathbf{1}_{\tau\leq T} | \mathcal{F}_T] | \mathcal{F}_t\} \\
= \; & \mathbb{E}^{\mathbb{Q}}\{\mathbf{1}_{\tau>t} \mathbb{E}^{\mathbb{Q}}[e^{-\int_t^\tau r_s ds} Z_\tau \mathbf{1}_{\tau\leq T} | \mathcal{F}_T] | \mathcal{F}_t\} \\
= \; & \mathbb{E}^{\mathbb{Q}}\{\mathbf{1}_{\tau>t} \int_t^T e^{-\int_t^u r_s ds} Z_u \gamma_u e^{-\int_0^u \gamma_s ds} du | \mathcal{F}_t\} \\
= \; & e^{-\int_0^t \gamma_s ds} \mathbb{E}^{\mathbb{Q}}\{\int_t^T e^{-\int_t^u R_s ds} Z_u \gamma_u du | \mathcal{F}_t\}
\end{aligned}$$

Credit Risk: Intensity-based models

Finally we simplify and recover:

$$\rightarrow \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{t<\tau\leq T} e^{-\int_t^T r_s ds} Z_\tau|\mathcal{G}_t] = \mathbf{1}_{\tau>t}\mathbb{E}^{\mathbb{Q}}\{\int_t^T e^{-\int_t^u R_s ds} Z_u\gamma_u du|\mathcal{F}_t\}$$

Pricing a zcb

The price of a zcb, with unit face value at maturity T, constant recovery rate RR, where the short rate process r_t and the hazard rate γ_t are independent is given as:

$$P(t,T) = \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\tau>T} e^{-\int_t^T r_s ds}|\mathcal{G}_t] + \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{t<\tau\leq T} RRe^{-\int_t^\tau r_s ds}|\mathcal{G}_t]$$

$$= \mathbf{1}_{\tau>t}\mathbb{E}^{\mathbb{Q}}[e^{-\int_t^T R_s ds}|\mathcal{F}_t] + \mathbf{1}_{\tau>t}\mathbb{E}^{\mathbb{Q}}[\int_t^T RR\gamma_s e^{-\int_t^s R_u du} ds|\mathcal{F}_t]$$

onto the observed sigma field $\mathcal{F}_t$.

Credit Risk: Multi-Factor Affine Intensity-based models

We assume to have a multi-factor affine structure for the process R_t. Nevertheless we recall that the term structure $\{p(t,T); 0 \leq t \leq T\}$ is said to be multi-factor affine iff it has the form:

$$p(t,T) = F(t, r_t, T) = \prod_{i=1}^{n} \exp\{A^i(t,T) - B^i(t,T) r_t^i\}$$

where $A^i(t,T)$ and $B^i(t,T)$ are deterministic functions while r_t is a vector of diffusion processes, which depends on independent Brownian motions. The ATS is assured by the following parameterization of the drift and diffusion of r_t:

$$\begin{cases} \mu^i(t, r_t) = \beta^i(t) + \alpha^i(t) r_t^i \\ \sigma^i(t, r_t) = \sqrt{\xi^i(t) r_t^i + \psi^i(t)} \end{cases}$$

Credit Risk: Multi-Factor Affine Intensity-based models

Consequently the PDE can be decomposed into two backward SDEs, i.e.

$$\begin{cases} \partial_t A^i = \beta^i(t)B^i - \frac{\psi^i(t)}{2}(B^i)^2 \\ A^i(T,T) = 0 \end{cases}$$

$$\begin{cases} \partial_t B^i = -1 - \alpha^i(t)B^i + \frac{\xi^i(t)}{2}(B^i)^2 \\ B^i(T,T) = 0 \end{cases}$$

whose solutions allow to recover the zcb prices.

Multi-Factor Affine Intensity-based models: CIR dynamic

Now considering vector r_t composed by the short rate r_t and the stochastic hazard rate γ_t, which are both one-dimensional independent CIR processes. The zcb turns out to be:

$$\begin{aligned} P(t,T) &= \mathbf{1}_{\tau>t}\exp\{\sum_{i=1}^{2} A^i(t,T) - B^1(t,T)r_t - B^2(t,T)\gamma_t\} + \\ &\quad + \mathbf{1}_{\tau>t}\mathbb{E}^{\mathbb{Q}}\left[\int_t^T RR\gamma_s e^{-\int_t^s R_u du} ds | \mathcal{F}_t\right] \end{aligned}$$

$$\begin{aligned} A^i(t,T) &= \frac{2k^i\theta^i}{(\sigma^i)^2}\log\left\{\frac{2h^i\exp\{(k^i+h^i)(T-t)/2\}}{2h^i+(k^i+h^i)(\exp\{(T-t)h^i\}-1)}\right\} \\ B^i(t,T) &= \frac{2(\exp\{(T-t)h^i\}-1)}{2h^i+(k^i+h^i)(\exp\{(T-t)h^i\}-1)} \\ h^i &= \sqrt{(k^i)^2+2(\sigma^i)^2}, i=1,2 \end{aligned}$$

CDS pricing: intensity-based model with doubly stochastic default time

CDS is an OTC standardized insurance contract where two parties exchange cash flows conditioned to the happening of pre-specified default event of a specific entity different by the two parties of the CDS. One party (the protection buyer) pays constant periodic premiums x to the other party (the protection seller), then in case of default of the specified entity the protection seller will pay an amount equal to the loss given by default suffered by the protection buyer (LGD).

$$\begin{aligned} V_{buyer} &= \mathbb{E}^{\mathbb{Q}}\left[\sum_{k=1}^{n} e^{-\int_t^{t_k} r_u du} x(t_k - t_{k-1})\mathbf{1}_{\tau > t_k} | \mathcal{G}_t\right] \\ V_{seller} &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{\tau} r_u du} LGD\mathbf{1}_{t<\tau\leq T} | \mathcal{G}_t\right] \end{aligned}$$

CDS pricing: intensity-based model with doubly stochastic default time

Therefore, imposing the equivalence of the two legs at the inception, we recover the fair premium, i.e. the CDS spread under the background filtration $\mathcal{F}_t$:

$$
\begin{aligned}
x &= \frac{\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{\tau} r_u du} LGD \mathbf{1}_{t<\tau\leq T} | \mathcal{G}_t\right]}{\mathbb{E}^{\mathbb{Q}}\left[\sum_{k=1}^{n} e^{-\int_t^{t_k} r_u du}(t_k - t_{k-1})\mathbf{1}_{\tau > t_k} | \mathcal{G}_t\right]} \\
&= \frac{\mathbb{E}^{\mathbb{Q}}\left[\int_t^T e^{-\int_t^{s} R_u du} LGD \gamma_s ds | \mathcal{F}_t\right]}{\mathbb{E}^{\mathbb{Q}}\left[\sum_{k=1}^{n} e^{-\int_t^{t_k} R_u du}(t_k - t_{k-1}) | \mathcal{F}_t\right]} \\
&= \frac{\mathbb{E}^{\mathbb{Q}}\left[\int_t^T e^{-\int_t^{s} R_u du} LGD \gamma_s ds | \mathcal{F}_t\right]}{\sum_{k=1}^{n}(t_k - t_{k-1})\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{t_k} R_u du} | \mathcal{F}_t\right]}
\end{aligned}
$$

Related Models: Lando (1997) and Duffie-Singleton (1999)

- Lando (1997): interest rates and default processes are driven by a set of state variables Y_t that is a multidimensional stochastic process, then the random time becomes:

$$\tau = \inf\{t \geq 0 : \int_0^t \gamma(Y_s)ds \geq X\}$$

  where X is exponentially distributed.
- Duffie-Singleton (1999): they specified the hazard $\gamma(Y_t)$ as a linear combination of macroeconomic factors and firm specific ones (firm specific factors, common factors also macroeconomic ones, interest rates). The driving factors of the short rate are not the same state variables driving the common factors.

This kind of specification of the hazard process can be easily integrated with a process that model the transition risky factor, as we have selected the right process to represent the last one.

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Model's Assumptions

- The default process is modelled using a doubly stochastic intensity based model: the flexibility in the definition of the hazard rate allows to consider external factors affecting the ability of the firm in repaying its debt;
- The process bringing the information about the transition risk is the spot price of the emission allowances in the EU-ETS (so the model is restricted to the participants of that market): we measure it in term of marginal cost of CO2 abatement since higher abatement costs requires higher returns so more risks that influence the firm ability of generating sufficient revenues of paying back its obligations. Thus, in an efficient cap and trade system the spot price of the emission allowances, in equilibrium, is equal to the marginal abatement cost.
- It is assumed that the EU-ETS market is efficient, meaning that the spot price is equal in equilibrium to the marginal abatement costs of GHGs: this assumption justifies the chosen measure of transition risk

Model's Assumptions

Technical assumptions:

- The dynamics of the emission allowances of the spot price is a diffusion process without jumps, i.e.

$$dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t^1$$

- the intensity process is composed by a carbon component and a firm specific factor, but not the short rates (since nowadays they are negative and cannot be included into the dynamic of γ_t) . Hence the multi-factor hazard rate is a linear combination, i.e. $d\gamma_t = d\lambda_t + \beta dS_t$

Model's Assumptions

Technical assumptions:

- the factors (r_t, S_t, λ_t) of the hazard rate, where S_t stands for the allowances spot price while λ_t is the firm-specific factor of the hazard are assumed to be mutually independent and λ_t is assumed to be a CIR process, i.e.

$$d\lambda_t = k^2(\theta^2 - \lambda_t)dt + \sigma^2\sqrt{\lambda_t}dW_t^2$$

- the short rate is assumed to be a diffusion process, i.e.

$$dr_t = \mu(t, r_t)dt + \sigma(t, r_t)dW_t^3$$

- the recovery rate is supposed to be constant

Alternative Models for S_t

- Cai-Pan (2017): they find that the best fitting for their sample of observed emission allowances spot prices is a non central Chi-square process, i.e.

  $$dS_t = k^1(\theta^1 - S_t)dt + \sigma^1\sqrt{S_t}dW_t^1$$

  Possibile Cons: 1) the dynamic is written under the historical measure $\mathbb{P}$, hence the drift includes an hidden component which is the market risk premium; 2) a mean reversion to a long run average is a strong assumption for a market divided in several phases affected by regulatory changes.

Alternative Models for S_t

- Chang, Su-Sheng, Huang (2012): they assume a log-normal $S_t = e^{x_t}$, where the exponent is an OU process, i.e.

$$dx_t = k^1(\theta^1 - x_t)dt + \sigma^1 dW_t^1$$

$$S_t = \exp\left\{x_s e^{-k^1(t-s)} + \theta^1(1 - e^{-k^1(t-s)}) + \sigma^1 \int_s^t e^{-k^1(t-u)} dW_u^1\right\}$$

Possible Cons and comments: 1) the model is not compatible with an affine structure of S_t, hence the price of zcb and CDS spread is not closed; 2) the spot price can be expressed as a multi-dimensional diffusion process (with also correlated Brownian), i.e. $\log(S_t) = \sum_{i=1}^n x_i$.

Alternative Models for S_t

Let $\mathbf{x} = (x_1, x_2, x_3)$ and we assume to be interest on the cdf of their sum. To this aim we first recall the following useful result (Cherubini *et al.*, 2011) defining the C-convolution operator.

Proposition

Let x_1 e x_2 be two real-valued random variables on the same probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with corresponding copula $C_{x_1,x_2}(w, \lambda)$ and continuous marginals F_{x_1} and F_{x_2}. Then, $\forall t$ a.s.

$$F_{x_1+x_2}(t) = \int_0^1 D_1 C_{x_1,x_2}\left(w, F_{x_2}(t - F_{x_1}^{-1}(w))\right) dw,$$

where $D_1 C_{x_1,x_2}(u, v) = \frac{\partial}{\partial u} C_{x_1,x_2}(u, v)$.

Alternative Models for S_t

Therefore, iterating the convolution, we have:

$$F_{x_1+x_2+x_3}(t) = \int_0^1 D_1 C_{x_1+x_2,x_3}\left(w, F_{x_3}(t - F^{-1}_{x_1+x_2}(w))\right) dw,$$

This copula-based approach allows to generalize Chang, Su-Sheng, Huang (2012) model to different dynamics of $\mathbf{x}$, beyond the gaussian one, including possible tail dependences. Nevertheless the computation must refer to numerical methods.

Alternative Models for S_t

- Daskalakis, Psychoyios and Markellos (2009): like a commodity price, under the $\mathbb{Q}$ the dynamics of S_t is a GBM. i.e.

$$dS_t = r_t S_t dt + \sigma^1 S_t dW_t^1$$

$$\to S_t = S_s \exp\left\{ \int_s^t r_u du - \frac{(\sigma^1)^2}{2}(t-s) + \sigma(W_t^1 - W_s^1) \right\}$$

Possible Cons and comments: the hazard process, which is a linear combination of specific factor and S_t is no more independent from the short interest rate dynamic; 2) to include a possible cost of carry (which seems to be realistic for the first period of every phase) it is enough to include a continuous and stochastic dividend in the dynamic of S_t. The stochastic convenience yield is hence modelled as a dividend for the emission right S_t.

Stochastic volatility model for S_t

One of the most restrictive technical assumption is the one about the constant volatility. A possible model to handle such feature of the data could be the Heston model, i.e.

$$\begin{cases} dS_t = r_t S_t dt + S_t \sqrt{\nu_t} dW_t^{11} \\ d\nu_t = \hat{k}(\hat{\nu} - \nu_t)dt + \eta\sqrt{\nu_t} dW_t^{12} \\ \mathbb{E}[dW_t^{11} dW_t^{12}] = \rho dt \end{cases}$$

whose solution, under the assumption of stochastic short-rate but independent from the stochastic factors affecting S_t, is

$$S_t = \exp\left\{ \int_s^t r_u - \frac{\nu_u}{2})du + \int_s^t \sqrt{\nu_u} dW_u^{11} \right\}$$

The dynamic of the hazard rate will be:

$$d\gamma_t = k^2(\theta^2 - \lambda_t)dt + \sigma^2\sqrt{\lambda_t} dW_t^2 + \beta r_t S_t dt + \beta S_t \sqrt{\nu_t} dW_t^{11}$$

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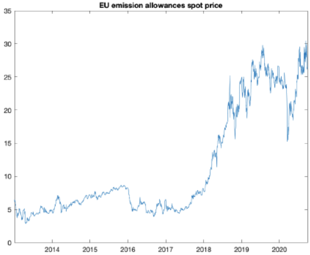
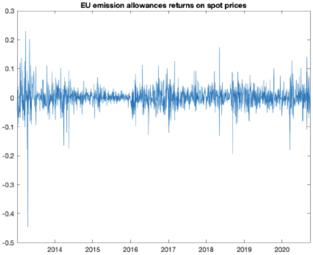
The emission allowances spot price

In the definition of the hazard process γ_t the main issue is represented by the choice of the dynamics driving the emission allowances spot price S_t.
The first step for doing it is to look at the data available on the market: here below the time series of the spot price and the daily returns starting from 2/01/2013 up to 28/09/2020.



(a) The carbon spot price



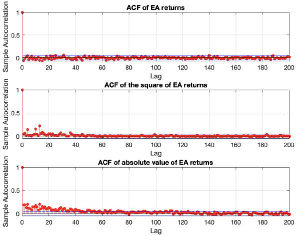
(b) Returns of carbon spot price

The emission allowances spot price

- in the third phase the carbon emission allowances spot price is a non stationary process; It is worthy to notice that from 2018 there is a increase of the spot price, that was due to the reform of the market stability reserve (MSR) which increased the efficiency of the market and whose consequence was an increase of the carbon spot price (the approach of Cai-Pan (2017) cannot be used since a mean reverting process is not supported by data of the third phase). A GBM could be a good choice.
- the returns of the carbon spot prices seem to be stationary with non constant volatility, suggesting the setting of a model with stochastic volatility for pricing derivatives written on emission allowances.

The emission allowances spot price

The empirical autocorrelation functions of the returns of the carbon prices (ACF of returns, of square returns, of abs value returns) show that returns are uncorrelated from the past but not independent; this evidence support the choice of a stochastic volatility model.

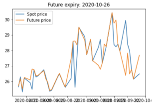


The emission allowances future price

The futures prices of the carbon rights and its relationship with spot prices will be analysed. The data are collected from Eikon-Thomson Reuters and they cover the following contracts:

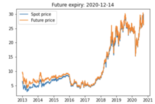
1. Futures expiring in 26/10/2020, 31/11/2020, 14/12/2020
2. Futures expiring in 22/03/2021, 28/06/2021, 27/09/2021, 20/12/2021
3. Futures expiring in 28/03/2022, 27/06/2022, 19/12/2022
4. Futures expiring in 18/12/2023
5. Futures expiring in 16/12/2024
6. Futures expiring in 15/12/2025
7. Futures expiring in 14/12/2026

The emission allowances future price

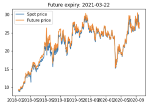


(a) Future-Spot expiry: 2020-10-26

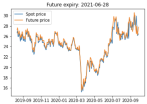
(b) Future-Spot expiry: 2020-11-30



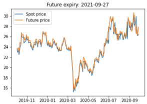
(a) Future-Spot expiry: 2020-12-14



(b) Future-Spot expiry: 2021-03-22

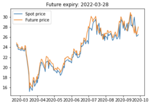


(a) Future-Spot expiry: 2021-06-28



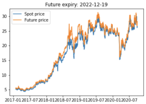
(b) Future-Spot expiry: 2021-09-27

(a) Future-Spot expiry: 2021-12-20



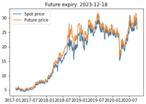
(b) Future-Spot expiry: 2022-03-28

(a) Future-Spot expiry: 2022-06-27

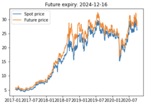


(b) Future-Spot expiry: 2022-12-19

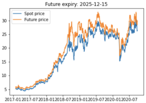
The emission allowances future price



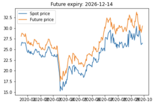
(a) Future-Spot expiry: 2023-12-18



(b) Future-Spot expiry: 2024-12-16



(a) Future-Spot expiry: 2025-12-15



(b) Future-Spot expiry: 2026-12-14

- Future price is usually greater than the spot one except for the first two futures, whose expiry (26/10/2020 and 30/11/2020) is close; here there is not an homogeneous behavior.
- The path of future price with long expiry date is similar to the spot price, just higher.

Model calibration

1. Collect the time series of the carbon spot price and analyse its features
2. Collect liquid derivatives written on carbon rights (futures or options if available)
3. Choose the model for the emission allowances spot price and calibrate the parameters based on the price of the derivatives collected before
4. Collect the CDS of firm that must buy them for emitting CO2 during its production process

Model calibration

1. With the calibrated dynamics for the carbon rights spot price, use the CDS spread representation to calibrate the parameters of the hazard rate (firm specific side, i.e. λ_t and the weight β) in order to reproduce the observed term structure for the CDS spreads
2. Finally after having calibrated the hazard process, simulate the survival/default probability of the firm. Such probabilities directly take into account the transition risk.

Calibration of Chang, Su-Sheng, Huang (2012) model

We consider the CSH (2012) model considering a 3-dimensional diffusion (with uncorrelated Brownian), i.e. $\log(S_t) = \sum_{i=1}^{3} x_i$, where

$$
\begin{aligned}
dx_t &= k(\theta - x_t)dt + \sigma dW_t \\
S_t &= \exp\Big\{ x_s^1 e^{-k^1(t-s)} + \theta^1(1 - e^{-k^1(t-s)}) + \sigma^1 \int_s^t e^{-k^1(t-u)} dW_u^{11} + \\
&\quad + x_s^2 e^{-k^2(t-s)} + \theta^2(1 - e^{-k^2(t-s)}) + \sigma^2 \int_s^t e^{-k^2(t-u)} dW_u^{12} + \\
&\quad x_s^3 e^{-k^3(t-s)} + \theta^3(1 - e^{-k^3(t-s)}) + \sigma^3 \int_s^t e^{-k^3(t-u)} dW_u^{13} \Big\}
\end{aligned}
$$

Calibration of Chang, Su-Sheng, Huang (2012) model

The future price can be recovered as follows (where we take advantage of the Laplace transform of a gaussian variable):

$$\mathbb{E}^{\mathbb{Q}}[S_t|\mathcal{F}_s] = \mathbb{E}_s^{\mathbb{Q}}\left[\exp\left\{\sum_{i=1}^{3} x_s^i e^{-k^i(t-s)} + \theta^i(1-e^{-k^i(t-s)}) + \right.\right.$$

$$\left.\left. +\sigma^i \int_s^t e^{-k^i(t-u)} dW_u^{1i}\right\}\right]$$

$$= \exp\left\{\sum_{i=1}^{3} x_s^i e^{-k^i(t-s)} + \theta^i(1-e^{-k^i(t-s)}) + \frac{(\sigma^i)^2}{4k^i}(1-e^{-k^i(t-s)})\right\}$$

Calibration of Chang, Su-Sheng, Huang (2012) model

We calibrate by the least square method, i.e. we recover the set of parameters which minimize the loss function given by the square difference of the market future prices and the theoretical future prices:

$$\min_{\Theta} \sum_{j=1}^{m} \sum_{i=1}^{n} (F_{t_i}(T_j) - \hat{F}_{t_i}(T_j))^2$$

where $\hat{F}_{t_i}(T_j) = \mathbb{E}^{\mathbb{Q}}[S_{T_j} | \mathcal{F}_{t_i}]$, $F_{t_i}(T_j)$ are the market quotes and Θ is the parameters' set.

Calibration of Chang, Sheng, Huang (2012) model

Here the calibration of CSH model with 1,2 and 3 factors respectively.

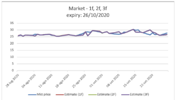
k	θ	σ
$3,6004 \cdot 10^{-3}$	7,9756	$3,391 \cdot 10^{-6}$

k_1	θ_1	σ_1
$8,877 \cdot 10^{-14}$	$6.1923 \cdot 10$	$4.5817 \cdot 10^{-1}$
k_2	θ_2	σ_2
$5.0172 \cdot 10^{-3}$	$-8.749 \cdot 10$	$6.04 \cdot 10^{-20}$

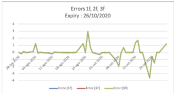
k_1	θ_1	σ_1
$1.114 \cdot 10^{-9}$	$1.045 \cdot 10$	$1.6193 \cdot 10^{-3}$
k_2	θ_2	σ_2
$7.8763 \cdot 10^{-3}$	$-3.5043 \cdot 10$	$4.3715 \cdot 10^{-8}$
k_3	θ_3	σ_3
$6.494 \cdot 10^{-15}$	7.6929	$2.94 \cdot 10^{-1}$

Calibration of Chang, Su-Sheng, Huang (2012) model

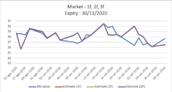
Comparison of the market quotes and the calibrated future prices.



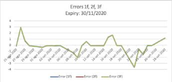
(a) Market future/Estimated future



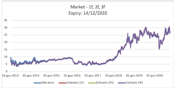
(b) Errors



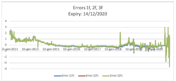
(a) Market future/Estimated future



(b) Errors

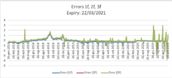


(a) Market future/Estimated future



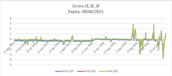
(b) Errors

(a) Market future/Estimated future



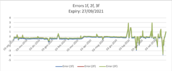
(b) Errors

(a) Market future/Estimated future



(b) Errors

(a) Market future/Estimated future

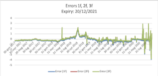


(b) Errors

Calibration of Chang, Su-Sheng, Huang (2012) model

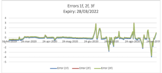
Comparison of the market quotes and the calibrated future prices.

(a) Market future/Estimated future



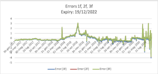
(b) Errors

(a) Market future/Estimated future



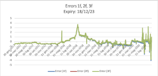
(b) Errors

(a) Market future/Estimated future



(b) Errors

(a) Market future/Estimated future



(b) Errors

(a) Market future/Estimated future

(b) Errors

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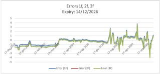
(b) Errors

Calibration of Chang, Su-Sheng, Huang (2012) model

Comparison of the market quotes and the calibrated future prices.

(a) Market future/Estimated future



(b) Errors

Data show that:

- it is clear that increasing the number of factors doesn improve the precision of the model

Calibration of Chang, Su-Sheng, Huang (2012) model

- even though the model is able to follow in general the movements of the future prices, the errors are not negligible. The futures with expiry 26/10/2020 and 30/11/2020 are not well represented by the model; the time series of those futures are quite short and it might be why the estimated parameters are not able to well represent such time series. On the other hand for the other futures the model is able to catch the movements of the prices.
- For the longer time series it is possible to observe a systematic peak in the error plots for the 2018; it might be due to the fact that the CSH model is not able to catch a steep increment as the one observed in 2018.
- The source of errors is mainly due to the assumption of constant volatility.

Final Remarks

The aim of the lecture is to proposed a method for directly integrating the transition risk into a credit risk model, hence we need a model for the credit risk and a measure for the transition risk.

- The chosen credit risk model has been the stochastic hazard rate approach which turned out be flexible enough for the scope
- The most problematic task was the choice of a variable representing the transition risk since a transition risk index nowadays doesn exist: the process bringing the information about the transition risk is assumed to be the emission allowances spot price in the EU market since 1) the EU scheme is a cap and trade system whose aim is the reduction of the CO2 and other greenhouse gases, and 2) for doing so in an efficient cap and trade system the spot price must be equal (theoretically speaking) to the marginal abatement cost which is a measure of the transition risk (if such costs are high then the expected return for such investments must be high and so the risk associated is high as well).

Final Remarks

The proposed model resulted by an hazard rate which is a linear combination of a firm specific component modelled as a CIR process (single or multi-factor) and a dynamics of the emission allowances spot price.

- The weak point of this approach is the assumption that the EU emission trading scheme is efficient and this is a critical task since in the first two phases it wasn't. Nevertheless the third phase is becoming increasingly efficient and such trend is most likely be kept even in the fourth phase (which started in 2021); the efficiency is crucial since it allows to say that the spot price is a good proxy of the marginal abatement cost.
- We devoted some time to a brief analysis of the emission allowances market in order to suggest a possible specification for the carbon spot price dynamics; we observed that stochastic volatility assumption seems to be a necessary condition to be well suited for the job.

Table of Contents

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Thank you very much!

For any question, feel free to ask or contact `silvia.romagnoli@unibo.it`


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